Comment on Coleman's theory of gravitation

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Abstract. The similarity of Coleman's theory to a special case of a class of theories presented by Papapetrou is discussed.

In a recent paper, Coleman (1971) presented a Lorentz-covariant theory of gravitation in which the 'metric' was given by

$$\mathrm{d}\sigma^2 = \mathrm{e}^{2\phi}\mathrm{d}t^2 - \frac{\mathrm{e}^{-2\phi}}{c^2}(\mathrm{d}\boldsymbol{r}\,.\,\mathrm{d}\boldsymbol{r}). \tag{1}$$

Trajectories of test particles are geodesics in this metric and ϕ is a solution of

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = 0. \tag{2}$$

The two-fold purpose of this note is to point out (i) that Coleman has discovered a variant of a class of theories which are the intellectual property of Papapetrou and (ii) to make some comments on the theory as presented.

Papapetrou (1954a, 1954b) sought to construct a generally covariant theory of gravitation which would be mathematically more tractable than the Einstein theory. His approach was to postulate the existence of at least one coordinate system in which the metric tensor would assume the simple form

$$\mathrm{d}s^2 = \mathrm{e}^{-\varphi}(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) + \mathrm{e}^{\chi}\mathrm{d}t^2. \tag{3}$$

In this coordinate system φ and χ were assumed to describe a pure gravitational field. In coordinate systems in which this canonical form did not obtain it would be presumed that inertial as well as pure gravitational forces were present. Field equations were obtained by means of the customary prescription for generally covariant theories. The latter part of the second paper contains an initial discussion of a further specialization obtained by stipulating that $\varphi + \chi = 0$.

In a third paper Papapetrou (1954c) explored this specialization in some detail. For the field equation he obtained.

$$\varphi_{,ii} + 3\mathrm{e}^{2\varphi}\varphi_{,00} + 3\mathrm{e}^{2\varphi}(\varphi_{,0})^2 = -\kappa(\zeta_0{}^0 - \zeta_i{}^i). \tag{4}$$

The right hand side vanishes in regions where the stress–energy tensor vanishes.

The complete identity of Coleman's theory with that of Papapetrou in the static case is manifest.

Coleman suggests that his theory has some similarity to that of Whitehead (1922). It is not clear what this similarity is. The only postulate in common is that test particles will move on geodesics in some 'dynamical' metric. Whitehead's theory has no field equations and the metric is determined by prescription in terms of the source distribution of charge and mass. It is essentially an action at a distance theory.

Coleman's theory does have field equations but has not specified how the source distribution enters. Presumably, it is to be specified in terms of a scalar constructed from the stress–energy tensor after the manner of Nordström (1913).

References

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Concerning Rao's rule

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Abstract. It is shown in a short note, that the expression derived by Mathur, Gupta and Sinha in their article entitled: 'Theoretical derivation of Wada's and Rao's relations', has been known for a long time. A short review of literature enclosed, makes possible the orientation in some problems related to the derivation and application of Kuczera's equation.

An article of Mathur *et al* (1971) has appeared as an attempt at a theoretical explanation of Rao's and Wada's relations. The authors arrived at the expression

$$2\alpha_c = -\{K(\nu+2)-2\}\alpha_v \tag{1}$$

in which

$$\alpha_c = \frac{1}{c}\frac{\mathrm{d}c}{\mathrm{d}T} \qquad \alpha_v = \frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}T} \qquad K \simeq 1.$$

c is a sound velocity and ν is an exponent in the potential of the molecular interaction

$$\varphi = -\alpha v^{-\mu} + \beta v^{-\nu}.$$

Equation (1) makes it possible to calculate the exponent ν in the Lennard–Jones formula. I should like to point out, that this relation has been known for a long time and in general it is known as Kuczera's formula (Kuczera 1957, 1958 and Kuczera